SMALL SCALE MODELS

AND

SIMILITUDE THEORY

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SIMILITUDE CONDITIONS

AND DIMENSIONAL ANALYSIS

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SIMILITUDE CONDITIONS AND DIMENSIONAL ANALYSIS

CHOOSE

- Geometry
- Materials
- Loading / actions intensity
- Load / action application history

EXTRAPOLATION OF RESULTS TO PROTOTYPE

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SIMILITUDE CONDITIONS AND DIMENSIONAL ANALYSIS

PHYSICS LAWS ARE HOMOGENEOUS

In physics laws, dimensions are coherent – the same in all equation terms.

Empirical laws may not be homogeneous; the units of the intervening quantities must be defined.

A PHYSICS LAW IS EXPRESSED BY PHYSICAL QUANTITIES WITH CERTAIN DIMENSIONS

- Length, L
- Force, F
- Time, t
- Temperature, θ
- Moment, FL
- Velocity, LT^{-1}
- Acceleration, LT^{-2}
- Stress, FL^{-2}
- Linear thermal dilat. coeff., θ^{-1}
- Strain, 1 (dimensionless)

Dimensions are expressed in certain units.

Ex.: Force (dimension) may be expressed in Newton (unit).

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SIMILITUDE CONDITIONS AND DIMENSIONAL ANALYSIS

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Empirical laws may not be homogeneous; in such case, the units of the intervening quantities must be previously defined.

Hazen-Williams empirical relationship (curve fitting) for the evaluation of pressure drop caused by friction in pipe flow

SI units [edit]

When used to calculate the head loss with the International System of Units, the equation becomes:[12]

$$S = \frac{h_f}{L} = \frac{10.67\,Q^{1.852}}{C^{1.852}\,d^{4.8704}}$$

where:

- S = Hydraulic slope
- h_f = head loss in meters (water) over the length of pipe
- L = length of pipe in meters
- Q = volumetric flow rate, m^3/s (cubic meters per second)
- C = pipe roughness coefficient
- d = inside pipe diameter, m (meters)

Note: pressure drop can be computed from head loss as h_f × the unit weight of water (e.g., 9810 N/m^3 at 4 deg C)

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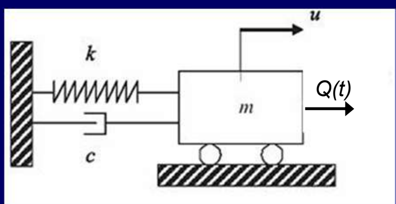
SIMILITUDE CONDITIONS AND DIMENSIONAL ANALYSIS

PHYSICS LAWS ARE HOMOGENEOUS

In physics laws, dimensions are coherent – the same in all equation terms.

Empirical laws may not be homogeneous; the units of the intervening quantities must be defined.

Differential equation of a 1 d.o.f. oscillator with damping
Physics law, coherent dimensions



$$F = ma$$

$$-K\,u - C\,u' + Q(t) = m\,u''$$

$$M\,u'' + C\,u' + K\,u = Q$$

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FUNDAMENTAL QUANTITIES

Static mechanical phenomena (2 fundamental quantities)
- Force, F
- Length, L

Dynamic mechanical phenomena (3 fundamental quantities)
- Force, F
- Length, L
- Time, t

Static mechanical phenomena with thermal variations (3 fundamental quantities):
- Force, F
- Length, L
- Temperature, θ

Dynamic mechanical phenomena with thermal variations (4 fundamental quantities):
- Force, F
- Length, L
- Time, t
- Temperature, θ

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SMALL SCALE MODELS AND SIMILITUDE THEORY

CONDIÇÕES DE SEMELHANÇA E ANÁLISE DIMENSIONAL

Buckingham theorem (Π theorem)

If a physical phenomenon involves <u>n</u> quantities, generically indicated as Ai (i=1,2,...,n), the functional relationship that rules the phenomenon, F(A1, A2 ..... , An) = 0, may be expressed by a relationship between (n-q) dimensionless parameters, $\Phi(\Pi 1, \Pi 2 \; ..... , \Pi n\text{-}q) = 0$, in which <u>q</u> is the number of physically (dimensionally) independent quantities that are involved in the phenomenon (q = 2 in static phenomena without ΔT; q = 3 in dynamic phenomena without ΔT, ...).

If the set of quantities Ai is reordered in such a manner that the fundamental quantities (those whose scales are chosen), X, Y, Z, correspond to An-2, An-1, An, than the dimensionless parameters Πi (with i=1, n-3) are in the form

$$\Pi i = \frac{A_i}{X^{\alpha_i} Y^{\beta_i} Z^{\gamma_i}} \qquad (1)$$

Exponents α_i, β_i e γ_i are determined by making Πi dimensionless.

The <u>similitude between a prototype and its model</u>, for the set of quantities (A_i) involved in the studied phenomenon, implies that, for all those quantities,

$$(\Pi i)_m = (\Pi i)_p \qquad (2)$$

From (1) and (2) the <u>scale</u> λ_{Ai} may be obtained for any quantity A_i by:

$$\lambda_{Ai} = (A_i)m / (A_i)p = (\lambda_X)^{\alpha_i} (\lambda_Y)^{\beta_i} (\lambda_Z)^{\gamma_i} \qquad (3)$$

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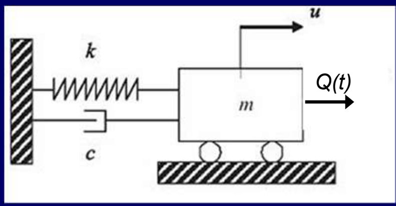
SIMILITUDE CONDITIONS AND DIMENSIONAL ANALYSIS



$$F = ma$$

$$-K\,u - C\,u' + Q(t) = m\,u''$$

$$M\,u'' + C\,u' + K\,u - Q = 0$$

$$M\,u'' + C\,u' + K\,u - Q = 0$$

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SMALL SCALE MODELS AND SIMILITUDE THEORY

Obtaining α_i, β_i, γ_i

Dimensionless parameters Π_i

$$\Pi_i = \frac{A_i}{X^{\alpha_i} Y^{\beta_i} Z^{\gamma_i}} \qquad \text{is dimensionless}$$

$$\frac{A_i(L^{l_{A_i}} F^{f_{A_i}} T^{t_{A_i}})}{X(L^{l_X} F^{f_X} T^{t_X})^{\alpha_i}\; Y(L^{l_Y} F^{f_Y} T^{t_Y})^{\beta_i}\; Z(L^{l_Z} F^{f_Z} T^{t_Z})^{\gamma_i}} \qquad \text{is dimensionless}$$

$$L^{l_{A_i}} F^{f_{A_i}} T^{t_{A_i}} = (L^{l_X} F^{f_X} T^{t_X})^{\alpha_i} (L^{l_Y} F^{f_Y} T^{t_Y})^{\beta_i} (L^{l_Z} F^{f_Z} T^{t_Z})^{\gamma_i}$$

$$L^{l_{A_i}} F^{f_{A_i}} T^{t_{A_i}} = L^{\alpha_i l_X + \beta_i l_Y + \gamma_i l_Z}\; F^{\alpha_i l_X + \beta_i l_Y + \gamma_i l_Z}\; T^{\alpha_i l_X + \beta_i l_Y + \gamma_i l_Z}$$

$$\begin{cases} l_{A_i} = \alpha_i l_x + \beta_i l_Y + \gamma_i l_Z \\ f_{A_i} = \alpha_i f_x + \beta_i f_Y + \gamma_i f_Z \\ t_{A_i} = \alpha_i t_x + \beta_i t_Y + \gamma_i t_Z \end{cases}$$

$$\begin{bmatrix} l_X & l_Y & l_Z \\ f_X & f_Y & f_Z \\ t_X & t_Y & t_Z \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \\ \gamma_i \end{bmatrix} = \begin{bmatrix} l_{A_i} \\ f_{A_i} \\ t_{A_i} \end{bmatrix} \longrightarrow \begin{cases} \alpha_i = \ldots \\ \beta_i = \ldots \\ \gamma_i = \ldots \end{cases}$$

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Obtaining the scales for quantities A_i

Equality of dimensionless coefficients between model and prototype

$$(\Pi_i)_m = (\Pi_i)_p$$

$$\left(\frac{A_i}{X^{\alpha_i} Y^{\beta_i} Z^{\gamma_i}}\right)_m = \left(\frac{A_i}{X^{\alpha_i} Y^{\beta_i} Z^{\gamma_i}}\right)_p$$

$$\frac{(A_i)_m}{(A_i)_p} = \left(\frac{X_m}{X_p}\right)^{\alpha_i} \left(\frac{Y_m}{Y_p}\right)^{\beta_i} \left(\frac{Z_m}{Z_p}\right)^{\gamma_i}$$

$$\boxed{\lambda_{A_i} = \lambda_X^{\alpha_i} \lambda_Y^{\beta_i} \lambda_Z^{\gamma_i}}$$

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CONSEQUENCES OF BUCKINGHAM THEOREM

- Any phenomenon can be reproduced on a scale different from the real one, even if the laws of physics (algebraic expressions) governing it are not known (eg relationship between wind speed and drag force on a bridge deck), provided that the respective dimensionless coefficients have the same value in the model and in the prototype, i.e., as long as the scale ratios of all quantities involved in this phenomenon are respected.

- The number of scales that can be chosen is equal to the number of fundamental quantities (number of dimensionally independent quantities) involved in the phenomenon under study. In a dynamic test, for example, one can choose the geometric scale (L), the modulus of elasticity (E) (choice of material) and the frequency scale (f), while all other parameters (A_i) involved in the phenomenon have to respect the scales obtained by:

$$\lambda_{A_i} = (\lambda_L)^{\alpha_i} \, (\lambda_E)^{\beta_i} \, (\lambda_f)^{\gamma_i}$$

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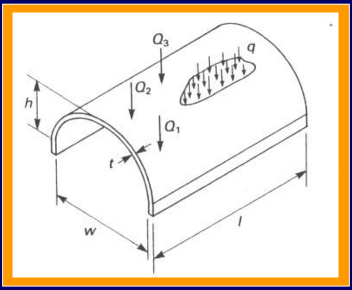
Example: Study of the static behaviour of a shell structure

Data:

- Static test in the elastic range
- Fundamental quantities: F - Force, L - Length; t - time is not involved
- Quantities to obtain: Stress σ, strain ε, displacement δ

Fundamental quantities (chosen):



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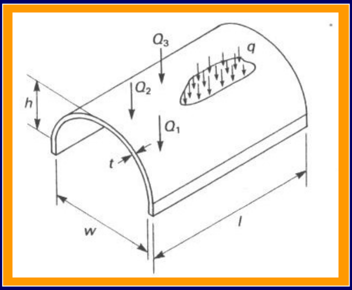
Example: Study of the static behaviour of a shell structure

Data:

- Static test in the elastic range
- Fundamental quantities: F - Force, L - Length; t - time is not involved
- Quantities to obtain: Stress σ, strain ε, displacement δ

Fundamental quantities (chosen):

- L, choosing the geometric scale



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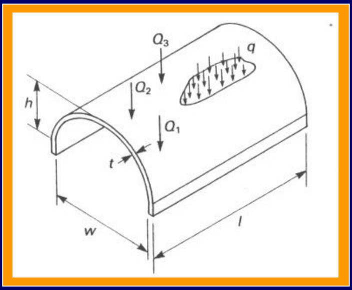
Example: Study of the static behaviour of a shell structure

Data:

- Static test in the elastic range
- Fundamental quantities: F - Force, L - Length; t - time is not involved
- Quantities to obtain: Stress σ, strain ε, displacement δ

Fundamental quantities (chosen):

- L, choosing the geometric scale
- E, choosing model's material



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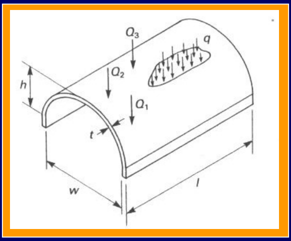
Exponents of different quantities written as a function of F and L

	t	L	w	h	E	ν	γ	q	Q	σ	ε
F	0	0	0	0	1	-	1	1	1	1	-
L	1	1	1	1	-2	-	-3	-2	0	-2	-



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Dimensionless coefficient

t	w	h	ν	γ	q	Q	σ	ε
$L^1 E^0$	$L^1 E^0$	$L^1 E^0$	$L^0 E^0$	$L^{-1} E^1$	$L^0 E^1$	$L^2 E^1$	$L^0 E^1$	$L^0 E^0$

Scales

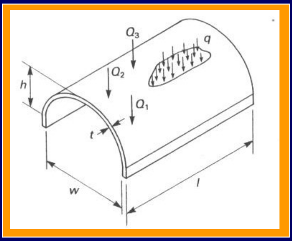
Geometry:

Linear dimension, displacement: (L): λ_L

Area (L^2): λ_L^2

Inertia (L^4): λ_L^4

Angular variation (-): 1



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Dimensionless coefficient

t	w	h	ν	γ	q	Q	σ	ε
$L^1 E^0$	$L^1 E^0$	$L^1 E^0$	$L^0 E^0$	$L^{-1} E^1$	$L^0 E^1$	$L^2 E^1$	$L^0 E^1$	$L^0 E^0$

Scales

Materials:

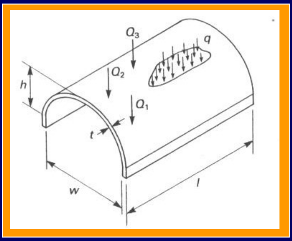
Young modulus (FL^{-2}): λ_E

Stress (FL^{-2}): λ_E

Poisson ratio (-): 1

Strain (-): 1

Specific weight (FL^{-3}): $\lambda_E \lambda_L^{-1}$



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Coeficientes adimensionais

$\frac{t}{L^1 E^0}$	$\frac{w}{L^1 E^0}$	$\frac{h}{L^1 E^0}$	$\frac{\nu}{L^0 E^0}$	$\frac{\gamma}{L^{-1} E^1}$	$\frac{q}{L^0 E^1}$	$\frac{Q}{L^2 E^1}$	$\frac{\sigma}{L^0 E^1}$	$\frac{\varepsilon}{L^0 E^0}$

Scales

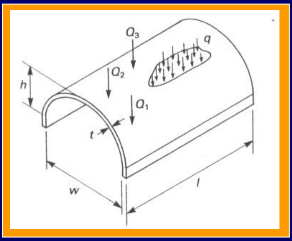
Loads, internal forces:

Conc. Load., shear force (F): $\lambda_E \lambda_L^2$

Pressure, distr. load (FL^{-2}): λ_E

Distributed load (FL^{-1}): $\lambda_E \lambda_L$

Moment (FL): $\lambda_E \lambda_L^3$



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NOTES ON SCALES

- The strain scale is unity because the quantity is dimensionless: equal strains in the model and prototype

- Poisson ratio scale is unity because the quantity is dimensionless: model material must have same ν of prototype

- The Young modulus is equal to the plane distributed load scale:
  $q_m = q_p E_m/E_p$ E_m low => q_m low

- The effect of self weight must be analysed considering that
  $\lambda_\rho = \lambda_E/\lambda_L$ => $\lambda_L = \lambda_E / \lambda_\rho$ <=> $\rho_m = \rho_p (\lambda_E/\lambda_L)$

  - if the material is the same λ_E=1, λ_ρ=1 => λ_L=1 (full scale)

  - to respect the self weight scale $\rho_m = \rho_p (\lambda_E/\lambda_L)$

    Ex. λ_E = 3 GPa (perspex) / 30 GPa (concrete) = 1/10;
    λ_L = 1/40 => ρ_m =4 ρ_p => application of additional loads to respect the self weight scale

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FABRICATION OF SMALL SCALE MODELS

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SMALL SCALE MODELS AND SIMILITUDE THEORY

ANALYSIS OF MECHANICAL PHENOMENA IN SMALL SCALE MODELS

FABRICATION OF THE SMALL SCALE MODEL

- SCALES SELECTION
- MATERIALS SELECTION
- DIMENSIONAL ANALYSIS

↓

EXPERIMENTAL ANALYSIS

- TEST SET UP
- ACTIONS APPLICATION (LOADS, ACCELERATIONS, WIND FLOW, WATER FLOW,...)
- CONTROL AND RECORDIND OF TEST PARAMETERS (LOADS, DISPLACEMENTS, ...)

↓

EXTRAPOLATION OF RESULTS TO PROTOTYPE

- DIMENSIONAL ANALYSIS / SCALES

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SMALL SCALE MODELS AND SIMILITUDE THEORY

WHEN TO USE SMALL SCALE MODELS?

Important and complex structures where analytical / numerical methods are not capable of satisfactorily reproducing the relevant phenomena.

Exs.:

- Post-elastic / near rupture phases of structural behaviour.
- Response to special actions – wind, non-linear behaviour of structures subjected to earthquakes, hydraulic flow, impacts, explosions, ...
- Before: analysis of hyperstatic structures, analysis of 3D structures.

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TYPES OF STRUCTURAL TESTS

- Static
- Dynamic
  - . Shaking table
  - . Dynamic actuators / reaction wall
  - . Jacks (slow response) / reaction wall (pseudo-dynamic tests)
- Wind tunnel
  - . Shape coefficients
  - . Aerodynamic (in)stability
- Others: Hydraulic, explosions, impacts (eg. crash tests involving planes), ...

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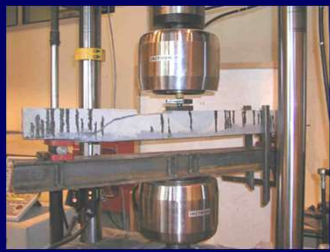
TYPES OF BEHAVIOUR

- Elastic (complex actions – wind, ...)
- Non linear (strength tests / rupture)



TEST SETUP

- Planning
- Analysis of similitude conditions (scales and materials)
- Fabrication process
- Loading system
- Measuring equipment

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MODEL FABRICATION

1) GEOMETRIC SCALE AND MATERIAL SELECTION

- Obtaining an adequate deformability
- Using an economic load application system
- Adequacy of the available measuring equipment
- Control / loading process
- Scale effects / non-dimensional parameters

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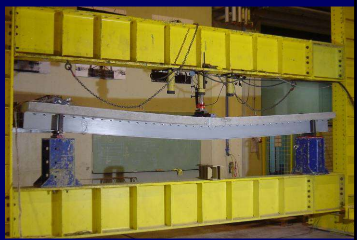
MODEL FABRICATION

2) MATERIAL CHARACTERISTICS

- Homogeneity
- Isotropy / anisotropy
- Poisson ratio (close to real)
- Easy to fabricate
- Economy



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MODEL FABRICATION

3) MATERIALS COMMONLY USED

- Plastics – elastic behaviour in the short term
- Admixtures of gypsum and diatomite – elastic behaviour, complex shapes (eg. dams)
- Steel – elastic and elastic-plastic behaviour
- Micro concrete (based on gypsum or Portland cement) – non-linear behaviour (material $\sigma-\varepsilon$ behaviour, cracking). Steel reinforcement in concrete.

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MODEL FABRICATION

PLASTICS (elastic behaviour at the short term)

- Easy to work
- Homogeneity
- Low E => lower loading (E ~ 3 GPa)
- Reasonable costs
- Creep / visocosity
- E varies with load application velocity
- High Poisson ratio (ν ~0,4)
- High linear expansion thermal coeficiente

PLASTICS

Termoplastic (plates, profiles; non mouldable):
Metil-metacrilato – Perspex, Plexiglas, Lucite
Policloreto de vinil – PVC

Thermosetting (mouldable):
Epoxy resins (fillers – adjusting ν, creep,...)
Polyester resins (idem)
Phenolic resins

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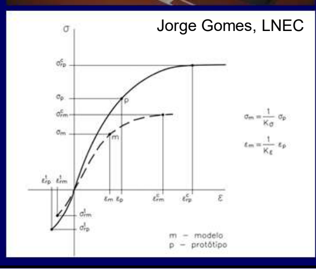
MODEL FABRICATION

ADMIXTURES OF GYPSUM AND DIATOMITE

- Easy to work (sculp)
- Homogeneity
- Low E, low strength => lower loading
- Allows complex shapes
- More fragile σ-ε behaviour than dam concrete
- Higher $\sigma_{tension}/\sigma_{compression}$ ratio than dam concrete



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MODEL FABRICATION

STEEL (elastic behaviour)

- Homogeneity
- High E => high loads

ALUMINIUM (elastic behaviour)

- Easy to work
- Homogeneity
- Lower E => lower loads (E = 70 GPa)
- ν = 0,45

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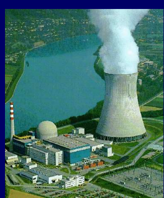
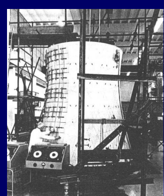
MODEL FABRICATION

MICRO CONCRETE (post elastic / rupture behaviour

- Easy to work
- Homogeneity
- Post-elastic behaviour
- Composition fine tuning / σ-ε similitude in tension (cracking) and compression (strength) – composition tests
- Concrete reinforcement and σ–ε similitude (ductility, atrength)
- Fabrication (reinforcement, formwork)





Binders: Portland cement or gypsum

Aggregates: Quartz powder, sand, fine aggregates (rolled stone pumice stone)

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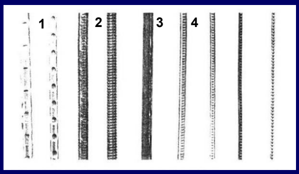
MODEL FABRICATION

MICRO CONCRETE – aspects to consider for reinforcement bars

- Identical σ–ε behaviour: yield and ultimate strength; shape of σ–ε diagram, ductility.

- Steel concrete adherence (rusty bars, hammered bars, corrugated bars, filleted bars,...). Difficult to evaluate at the prototype. Difficult to simulate. Pull-out tests. Tension tests on reinforced concrete ties – spacing between primary cracks should comply with geometric scale.

- Stirrups don't need to respect the similitude conditions related to adhesion to concrete.



1 – Hammered bar: different σ–ε curve
2 – Filleted bar: excessive adherence
3 – Plain bar: insufficient adherence
4 – Corrugated bar: adequate
5 – Rusty bar: irregular adherence

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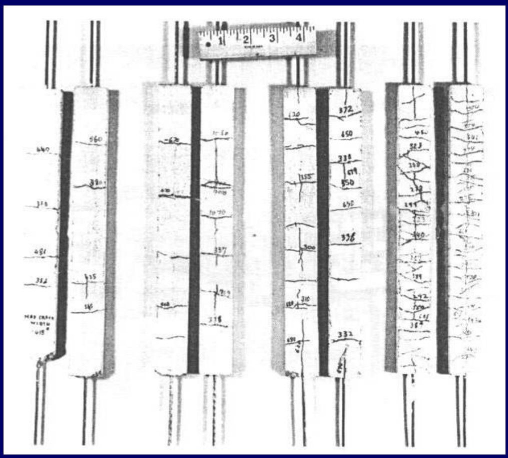
Steel-concrete adhesion

Tension tests in reinforced concrete ties



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MODEL FABRICATION

MICRO CONCRETE – aspects to consider for prestress

- Elements used: prestress wires; piano strings; helical threaded steel bars.

- Pre-tension vs. post-tension:
  - In **pre-tension**, **adherence** similitude is of utmost importance;
  - In **post-tension,** adequate **anchoring** systems must be considered.

- Prestress losses (short term and long term) must be adequately reproduced.

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SMALL SCALE MODELS AND SIMILITUDE THEORY

STRENGTH MODELS

- The model must present a similar behaviour until rupture / colapse, i.e., the σ–ε diagram must be identical

- It is usual to use concrete with the same ultimate strain ε_r (λ_ε=1) but with different strength σ_r ($\lambda_\sigma \neq 1$) which implies $\lambda_E = \lambda_\sigma \neq 1$ => it is not possible to use the same concrete and steel in the model and in the prototype.

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CHOOSING THE GEOMETRIC SCALE

- Very large models: High loads; more expensive ecquipment; more space needed.

- Very small models: Difficult fabrication; requires more accurate instruments; more difficult to respect similitude conditions (scale effects).

- The chosen scale often depends on the characteristics of the materials commercially available (eg thickness of plastic sheets) or on the model exequibility (minimum concrete thickness).

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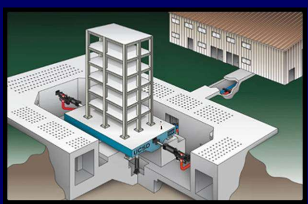
LOAD APPLICATION SYSTEMS

APPLIED LOADS

- Hydraulic jacks
- Screw jacks
- Sand bags
- Water
- …

SELF WEIGHT

- Applied loads
- Additional weights
- Dense materials (theoretical)
- …



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ANALYSIS OF RESULTS

THE QUALITY OF THE RESULTS DEPENDS ON:

- Material properties
- Quality of fabrication
- Errors in loading
- Measuring instruments

Errors between 10% to 15% are usually acceptable.

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SPECIAL CASES OF DYNAMIC PHENOMENON

$$F_{inércia} = ma = m\frac{dV}{dt} = m\frac{dV}{dt}\frac{ds}{ds} = m\frac{dV}{ds}\frac{ds}{dt} = m\frac{dV}{ds}V \rightarrow m\frac{V}{L}V = \rho L^3\frac{V}{L}V = \rho L^2 V^2$$

$$F_{gravítica} = mg = \rho L^3 g$$

$$F_{elástica} = K\,x = m\frac{K}{m}x = m\sqrt{\frac{K}{m}}^{2}x = m\,\omega^2 x = m\,(2\pi f)^2\,x \propto m\,f\,(f\,x) = \rho L^3\,f\,V$$

1 d.o.f. oscillator:

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$x = x\sin(\omega t)$$

$$V = \omega x\cos(\omega t) = 2\pi f x\cos(\omega t) \propto f x\cos(\omega t)$$

$$F_{viscosa} = \mu L V$$

In dynamic phenomena inertial forces always intervene F = ma

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SPECIAL CASES OF DYNAMIC PHENOMENON

1) Cauchy similitude: when elastic forces are relevant

Cauchy number $C_a = \frac{F_{inércia}}{F_{elástica}} = \frac{\rho L^2 V^2}{\rho L^3 f V} = \frac{V}{L f}$

Escalas fundamentais:

λ_L (escala geométrica)

λ_ρ (massa específica, escolha do material)

λ_V (velocidade)

$$(C_a)_m = (C_a)_p \Rightarrow \left(\frac{V}{L f}\right)_m = \left(\frac{V}{L f}\right)_p \Rightarrow \frac{V_m}{V_p} = \frac{L_m}{L_p}\frac{f_m}{f_p} \Rightarrow \lambda_V = \lambda_L \lambda_f \Rightarrow \lambda_f = \lambda_L^{-1}\lambda_V$$

Ex.: Cable stayed bridges under wind loading

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SPECIAL CASES OF DYNAMIC PHENOMENON

2) Froude similitude: when gravitic loads are relevant

Froude number $F_r = \sqrt{\frac{F_{inércia}}{F_{gravítica}}} = \sqrt{\frac{\rho L^2 V^2}{\rho L^3 g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$

Escalas fundamentais:

λ_L (escala geométrica)

$\lambda_g = 1$ (gravidade)

λ_ρ (massa específica, escolha do material)

$$(F_r)_m = (F_r)_p \Rightarrow \left(\frac{V}{\sqrt{Lg}}\right)_m = \left(\frac{V}{\sqrt{Lg}}\right)_p \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}\sqrt{\frac{g_m}{g_p}} \Rightarrow \lambda_V = \lambda_L^{1/2}\lambda_g^{1/2} \Rightarrow \lambda_V = \lambda_L^{1/2}$$

Escala de tempos

$$\frac{t}{L^{\alpha_t} g^{\alpha_t} \rho^{\alpha_t}} = \frac{t}{L(L)^{\alpha_t}\, g(Lt^{-2})^{\beta_t}\, \rho(mL^{-3})^{\gamma_t}} \text{ adimensional} \Rightarrow \alpha_t = 1/2 \quad \beta_t = -1/2 \quad \gamma_t = 0$$

$$\lambda_t = \lambda_L^{1/2}\lambda_g^{1/2}\lambda_\rho^0 = \lambda_L^{1/2}\, 1^{1/2}\, 1^0 = \lambda_L^{1/2}$$

Ex.: Suspension bridges (stiffness depends on gravitic loads) under wind loading.

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SPECIAL CASES OF DYNAMIC PHENOMENON

3) Reynolds similitude: when viscous forces are relevant

Reynolds number $R_e = \frac{F_{inércia}}{F_{viscosa}} = \frac{\rho L^2 V^2}{\mu L V} = \frac{\rho L V}{\mu}$

Escalas fundamentais:

λ_L (escala geométrica)

λ_ρ (massa específica, escolha do material fluido)

λ_μ (viscosidade, escolha do material fluido)

$$(R_e)_m = (R_e)_p \Rightarrow \left(\frac{\rho L V}{\mu}\right)_m = \left(\frac{\rho L V}{\mu}\right)_p \Rightarrow \frac{\rho_m}{\rho_p}\frac{L_m}{L_p}\frac{V_m}{V_p} = \frac{\mu_m}{\mu_p} \Rightarrow \lambda_\rho \lambda_L \lambda_V = \lambda_\mu \Rightarrow \lambda_V = \lambda_\mu \lambda_L^{-1} \lambda_\rho^{-1}$$

$\lambda_\mu = \lambda_\rho = 1$ (mesmo fluido) $\Rightarrow \lambda_V = \lambda_L^{-1}$ Ex. $\lambda_L = 1/50 \Rightarrow \lambda_V = 50/1$ (inviável)

Ex.: Wind tunnel. In structures with sharp edges, if $R_e > 300$ to 1000, the Reynolds similitude has not to be obeyed, as the locations of the boundary layer separation are known (they do not depend on Reynolds number).

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SPECIAL CASES OF DYNAMIC PHENOMENON

Dynamic phenomena where gravitic and viscous forces are relevant: Ex. Suspension bridges with aerodynamic shaped deck.

Simultaneously follow Froude and Reynolds similitude

Froude $\lambda_V = \lambda_L^{1/2}$

Reynolds $\lambda_V = \lambda_L^{-1}$

$\Rightarrow \lambda_L = 1$ Full scale

Conclusion: it is not possible to simultaneously follow Froude and Reynolds similitude in small scale models.

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SMALL SCALE MODELS AND SIMILITUDE THEORY

SPECIAL CASES OF DYNAMIC PHENOMENON

Simultaneously follow Froude and Reynolds similitude

Froude $\lambda_V = \lambda_L^{1/2}$

Reynolds $\lambda_V = \lambda_L^{-1}$

$\Big\}$ $\lambda_L = 1$ Full scale

Why does this happen? When imposing Froude and Reynolds similitude, 3 unity scales were implicitly chosen, thus those quantities were taken as the fundamental ones. Thus, the remaining scales, including the geometric scale, are also unitary.

$$\lambda_g = 1 \text{ (escala das acelerações – gravidade)}$$
$$\lambda_\rho = 1 \text{ (escala da massa específica – fluido)}$$
$$\lambda_\mu = 1 \text{ (escala da viscosidade – fluido)}$$

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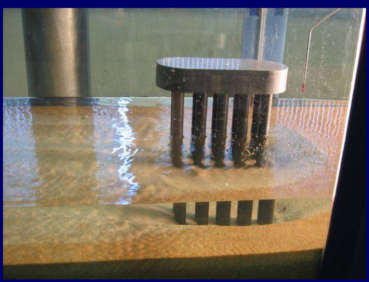
SMALL SCALE MODELS AND SIMILITUDE THEORY

VISIT TO THE LABORATORY OF HYDRAULICS

- Wave channel: project to use wave energy for electricity generation (Eng.º Miguel Lopes).
- Free surface channel: Analysis of desiltation in bridge piles.



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